Math 53, Discussions 116 and 118 **2022-09-09**

Conic sections and intro to 3D and vectors

Questions

Question 1. What does the equation $y^2 = 4$ describe in $\mathbb{R}^2$? What about $\mathbb{R}^3$?

Question 2. If $\mathbf{r} = \langle x, y \rangle$, $\mathbf{a} = \langle a_1, a_2 \rangle$, and $\mathbf{b} = \langle b_1, b_2 \rangle$ (where a_1, a_2, b_1, b_2 are constants), expand out the equation

$$(\mathbf{r} - \mathbf{a}) \cdot (\mathbf{r} - \mathbf{b}) = 0$$

and say what kind of shape it is.

Question 3. Can you express the magnitude (length) of a vector $\mathbf{v}$ in terms of the scalar (dot) product?

Question 4. Do the surfaces defined by the equations

$$x^2 + y^2 + (z-1)^2 = 25$$

and

$$x^2 + y^2 + z^2 = 9$$

intersect?

Question 5. Suppose that H_1 and H_2 are two planes in $\mathbb{R}^3$ (3-dimensional space). Which of the following might be the intersection $H_1 \cap H_2$? There are multiple correct answers.

(a) A plane.
(b) A line.
(c) A point.
(d) Empty (the planes don't intersect).

Question 6. Identify the following shapes in $\mathbb{R}^2$. Just a simple verbal description is fine.

(a) $4x^2 - 12x - 9y^2 - 6y + 7 = 0$
(b) $4x^2 - 12x - 9y^2 - 6y + 8 = 0$
(c) $4x^2 - 12x - 9y^2 - 6y + 9 = 0$

Question 7. Consider the line L with parametric equations

$$x = 3 + 3t, \qquad y = 2 - t, \qquad z = 5t$$

and the point $P(1, -2, 2)$. Find the point Q on the line L which minimizes the distance $|PQ|$, and say what this minimum distance is.